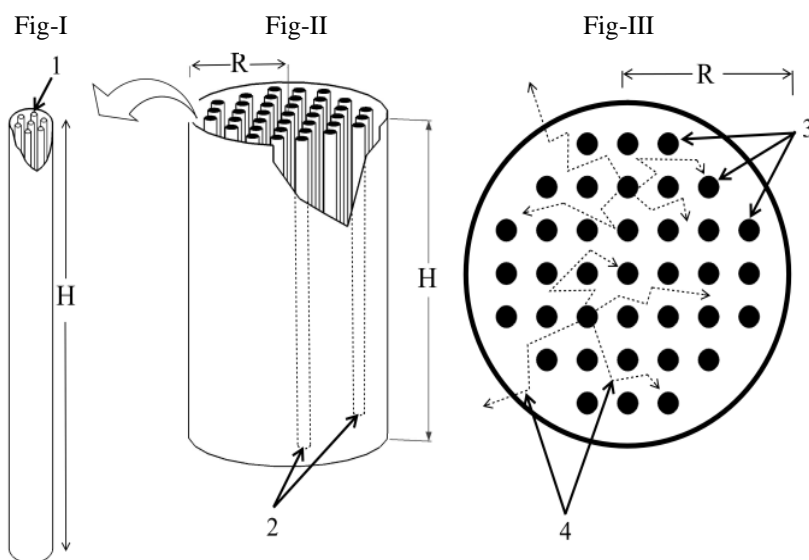


The Design of a Nuclear Reactor

(Total Marks: 10)

Uranium occurs in nature as  $\text{UO}_2$  with only 0.720% of the uranium atoms being  $^{235}\text{U}$ . Neutron induced fission occurs readily in  $^{235}\text{U}$  with the emission of 2-3 fission neutrons having high kinetic energy. This fission probability will increase if the neutrons inducing fission have low kinetic energy. So by reducing the kinetic energy of the fission neutrons, one can induce a chain of fissions in other  $^{235}\text{U}$  nuclei. This forms the basis of the power generating nuclear reactor (NR).

A typical NR consists of a cylindrical tank of height  $H$  and radius  $R$  filled with a material called moderator. Cylindrical tubes, called fuel channels, each containing a cluster of cylindrical fuel pins of natural  $\text{UO}_2$  in solid form of height  $H$ , are kept axially in a square array. Fission neutrons, coming outward from a fuel channel, collide with the moderator, losing energy and reach the surrounding fuel channels with low enough energy to cause fission (Figs I-III). Heat generated from fission in the pin is transmitted to a coolant fluid flowing along its length. In the current problem we shall study some of the physics behind the (A) Fuel Pin, (B) Moderator and (C) NR of cylindrical geometry.



Schematic sketch of the Nuclear Reactor (NR)

Fig-I: Enlarged view of a fuel channel (1-Fuel Pins)

Fig-II: A view of the NR (2-Fuel Channels)

Fig-III: Top view of NR (3-Square Arrangement of Fuel Channels and 4-Typical Neutron Paths).

Only components relevant to the problem are shown (e.g. control rods and coolant are not shown).

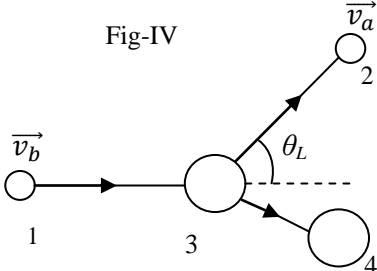
A Fuel Pin

Data for $\text{UO}_2$	1. Molecular weight $M_w = 0.270 \text{ kg mol}^{-1}$	2. Density $\rho = 1.060 \times 10^4 \text{ kg m}^{-3}$
	3. Melting point $T_m = 3.138 \times 10^3 \text{ K}$	4. Thermal conductivity $\lambda = 3.280 \text{ W m}^{-1} \text{ K}^{-1}$

A1	Consider the following fission reaction of a stationary $^{235}\text{U}$ after it absorbs a neutron of negligible kinetic energy. $^{235}\text{U} + ^1_0\text{n} \rightarrow ^{94}\text{Zr} + ^{140}\text{Ce} + 2 ^1_0\text{n} + \Delta E$ Estimate $\Delta E$ (in MeV) the total fission energy released. The nuclear masses are: $m(^{235}\text{U}) = 235.044 \text{ u}$ ; $m(^{94}\text{Zr}) = 93.9063 \text{ u}$ ; $m(^{140}\text{Ce}) = 139.905 \text{ u}$ ; $m(^1_0\text{n}) = 1.00867 \text{ u}$ and $1 \text{ u} = 931.502 \text{ MeV } c^{-2}$ . Ignore charge imbalance.	0.8
A2	Estimate $N$ the number of $^{235}\text{U}$ atoms per unit volume in natural $\text{UO}_2$ .	0.5
A3	Assume that the neutron flux density, $\phi = 2.000 \times 10^{18} \text{ m}^{-2} \text{ s}^{-1}$ on the fuel is uniform. The fission cross-section (effective area of the target nucleus) of a $^{235}\text{U}$ nucleus is $\sigma_f = 5.400 \times 10^{-26} \text{ m}^2$ . If 80.00% of the fission energy is available as heat, estimate $Q$ (in $\text{W m}^{-3}$ ), the rate of heat production in the pin per unit volume. $1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}$	1.2
A4	The steady-state temperature difference between the center ( $T_c$ ) and the surface ( $T_s$ ) of the pin can be expressed as $T_c - T_s = k F(Q, a, \lambda)$ , where $k = 1/4$ is a dimensionless constant and $a$ is the radius of the pin. Obtain $F(Q, a, \lambda)$ by dimensional analysis. Note that $\lambda$ is the thermal conductivity of $\text{UO}_2$ .	0.5
A5	The desired temperature of the coolant is $5.770 \times 10^2 \text{ K}$ . Estimate the upper limit $a_u$ on the radius $a$ of the pin.	1.0

**B The Moderator**

Consider the two dimensional elastic collision between a neutron of mass 1 u and a moderator atom of mass A u. Before collision all the moderator atoms are considered at rest in the laboratory frame (LF). Let  $\vec{v}_b$  and  $\vec{v}_a$  be the velocities of the neutron before and after collision respectively in the LF. Let  $\vec{v}_m$  be the velocity of the center of mass (CM) frame relative to LF and  $\theta$  be the neutron scattering angle in the CM frame. All the particles involved in collisions are moving at nonrelativistic speeds.

B1	<p>The collision in LF is shown schematically, where <math>\theta_L</math> is the scattering angle (Fig-IV). Sketch the collision schematically in CM frame. Label the particle velocities for 1, 2 and 3 in terms of <math>\vec{v}_b</math>, <math>\vec{v}_a</math> and <math>\vec{v}_m</math>. Indicate the scattering angle <math>\theta</math>.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center; margin-right: 20px;"> <p>Fig-IV</p>  </div> <div style="border: 1px solid black; padding: 5px; margin-left: 20px;"> <p style="text-align: center; margin: 0;"><i>Collision in the Laboratory Frame</i></p> <p style="margin: 0;">1-Neutron before collision</p> <p style="margin: 0;">2-Neutron after collision</p> <p style="margin: 0;">3-Moderator Atom before collision</p> <p style="margin: 0;">4-Moderator Atom after collision</p> </div> </div>	1.0
B2	Obtain $v$ and $V$ , the speeds of the neutron and moderator atom in the CM frame after collision, in terms of $A$ and $v_b$ .	1.0
B3	Derive an expression for $G(\alpha, \theta) = E_a/E_b$ , where $E_b$ and $E_a$ are the kinetic energies of the neutron, in the LF, before and after the collision respectively and $\alpha \equiv [(A - 1) / (A + 1)]^2$ .	1.0
B4	Assume that the above expression holds for D <sub>2</sub> O molecule. Calculate the maximum possible fractional energy loss $f_l \equiv \frac{E_b - E_a}{E_b}$ of the neutron for the D <sub>2</sub> O (20 u) moderator.	0.5

**C The Nuclear Reactor**

To operate the NR at any constant neutron flux  $\psi$  (steady state), the leakage of neutrons has to be compensated by an excess production of neutrons in the reactor. For a reactor in cylindrical geometry the leakage rate is  $k_1 [(2.405/R)^2 + (\pi/H)^2] \psi$  and the excess production rate is  $k_2 \psi$ . The constants  $k_1$  and  $k_2$  depend on the material properties of the NR.

C1	Consider a NR with $k_1 = 1.021 \times 10^{-2} \text{ m}$ and $k_2 = 8.787 \times 10^{-3} \text{ m}^{-1}$ . Noting that for a fixed volume the leakage rate is to be minimized for efficient fuel utilization, obtain the dimensions of the NR in the steady state.	1.5
C2	The fuel channels are in a square arrangement (Fig-III) with the nearest neighbour distance 0.286 m. The effective radius of a fuel channel (if it were solid) is $3.617 \times 10^{-2} \text{ m}$ . Estimate the number of fuel channels $F_n$ in the reactor and the mass $M$ of UO <sub>2</sub> required to operate the NR in steady state.	1.0